

APPLICATION OF MATHEMATICAL MODELING IN MECHANICS THROUGH AN EXAMPLE IN TECHNICAL SYSTEMS

Ivana Terzic¹ Jana Jovanovic² Snežana Gavrilovic³

Abstract: Mathematical models in mechanics enable the analysis and prediction of the behavior of physical systems by transforming complex processes into mathematical expressions. These models are crucial for the development of technology, engineering, and scientific theory. This paper presents a mathematical model of an electromagnetic switch, focusing on the mechanical subsystem that includes the spring and the anchor. The model describes the forces acting on the motion of the anchor, such as the electromagnetic force, spring force, friction, and pressure difference. Through simulations in Simulink, the factors influencing the response time of the switch were analyzed, such as the spring force and the *magnitude of the electromagnet's voltage*. *The paper highlights the importance of mathematical modeling in system optimization, as it enables a better understanding and control of complex technical problems.*

Key words: mathematical modeling, mathematical model, mechanics, electromagnetic switch, technical system.

1. INTRODUCTION

Mathematical models in mechanics play a crucial role as they enable the analysis, prediction, and understanding of the behavior of physical systems. They allow complex physical processes and systems to be represented by mathematical expressions and formulas that can be analyzed, solved, simulated on a computer, and used for predicting behavior. Mechanics, which studies the motion of bodies and the forces acting on them, uses mathematical models to describe, analyze, and solve real-world problems.

Many publications address this topic. The fundamental mathematical models of classical mechanics, commonly used to describe the motion of real mechanical systems, are discussed in [1]. Mathematical models that can be expressed as systems of ordinary differential equations and address the solution of various real-world problems are explored by the authors in [2]. The publication [3] covers three closely related areas: mathematical modeling in mechanics, numerical analysis, and optimization methods. The authors in [4] focus on mathematical modeling and simulation.

2. SIGNIFICANCE OF MATHEMATICAL MODELS IN MECHANICS

The significance of mathematical models in mechanics, based on [5, 6, 7, and 8], can be seen in several aspects:

- **Accurate description of physical laws:** Mathematical models allow physical laws, such as Newton's laws of motion or the laws of conservation of energy and momentum, to be written as mathematical expressions. For example, the motion of a body under the influence of force is described by differential equations that include forces, accelerations, and the motion of objects. Without such mathematical expressions, it would be impossible to accurately analyze and predict the behavior of bodies in mechanical systems.
- **Understanding and analysis of complex systems:** Many systems in mechanics are too complex to be analyzed directly or experimentally, such as systems in fluid dynamics or thermodynamics. Mathematical models allow for the analysis of these systems using numerical methods and simulations, such as studying the motion of fluids under different conditions, which helps us understand phenomena like turbulence or flow through irregular shapes.

¹Master of Mechanical Engineering, Faculty of Engineering Sciences, University of Kragujevac, Kragujevac, Serbia, ivana.aa.terzic@gmail.com

²Master of Industrial Engineering, Academy of Professional Studies Sumadija, College in Trstenik, Trstenik, Serbia, jance99@gmail.com

³PhD of Computer Science, Academy of Professional Studies Sumadija, College in Trstenik, Trstenik, Serbia, sgavrilovic@asss.edu.rs

- **Optimization and design:** In engineering, mathematical models assist in optimizing the design of objects such as bridges, buildings, or airplanes. These models enable the analysis of the behavior of structures under various conditions, such as weight or weather conditions, and help design objects that are safer, more efficient, and cost-effective. For example, in aerodynamics, mathematical models help design airplanes with minimal air resistance.
- **Simulations and experiments:** Mathematical models enable simulations that replicate real conditions. Simulations help study the behavior of systems even when experimental research is expensive or difficult. For example, crash simulations in car accidents use mathematical models of motion and collisions to assess vehicle safety and develop safety systems like airbags.
- **Solving real-world problems:** In many practical situations, mechanical problems are complex. For instance, to predict the motion of objects in mechanical systems such as satellites or industrial machines, mathematics is used to solve differential equations that describe the dynamics of the system. These models help predict system behavior and address technical challenges in the real world.
- **Verification and validation of models:** Mathematical models enable the verification of the accuracy of theoretical predictions through experimental measurements and data. Based on this data, we analyze whether the predictions are accurate and if modifications to the model are necessary. Experiments often help verify the accuracy of the model, while mathematical tools facilitate the analysis and interpretation of these results.
- **Development of new theories:** Mathematical models are not only used to solve existing problems but can also lead to the development of new theories. For example, the theory of relativity and many modern theories in mechanics were developed with the help of mathematics. Similarly, theories describing mechanical systems under extreme conditions, such as black holes or quantum mechanics, use sophisticated mathematical models.

3. EXAMPLE OF ELECTROMAGNETIC SWITCH MODELING

3.1. Mathematical Modeling

The procedure of mathematical modeling includes defining the problem, using the appropriate laws, and representing the model in an appropriate form (differential equations, simulation schemes for computers). The system can be studied in several ways:

- By experimentation,
- By mathematical analysis of the system's operation,
- By modeling and simulation.

Modeling represents the link between the real system and the model (Fig. 1), simulation is the link between the model and the computer, while the model is an abstract representation of the system. It describes part of the structure or behavior of the real system, containing selected elements and characteristics of the system that are essential for research.

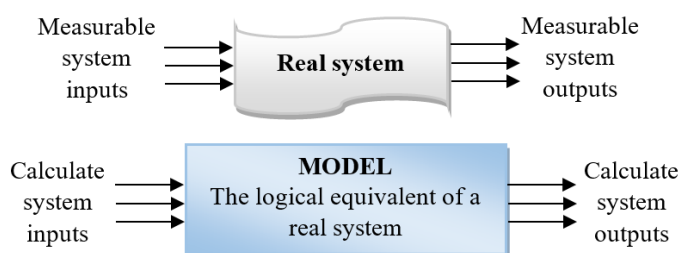


Figure 1 – System modeling

3.2. Mathematical Model of a Mechanical System

Mathematical models that describe phenomena in a pneumatic switch are presented in [9].

The operating principle of a pneumatic switch with two piston positions and three ports is shown in Figure 2. When the switch is off (Fig. 2.a), there is a connection between the output of the switch and atmospheric pressure. When the switch is on (Fig. 2.b), this connection is interrupted, and a new connection is established between the working pressure and the output.

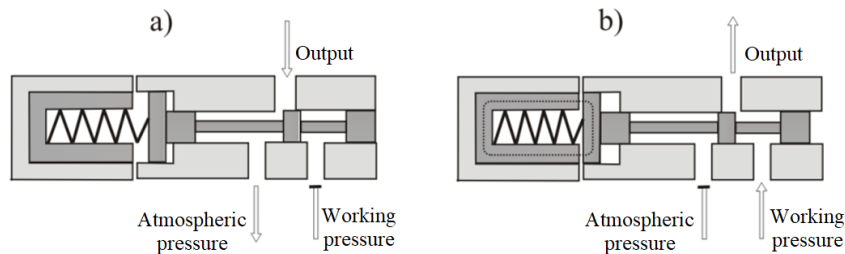


Figure 2 – Operating principle of the pneumatic switch

The model of the electromagnetic switch consists of three subsystems, and their mutual connections are shown in Figure 3. In this example of mathematical modeling, special attention is focused on the mechanical subsystem.

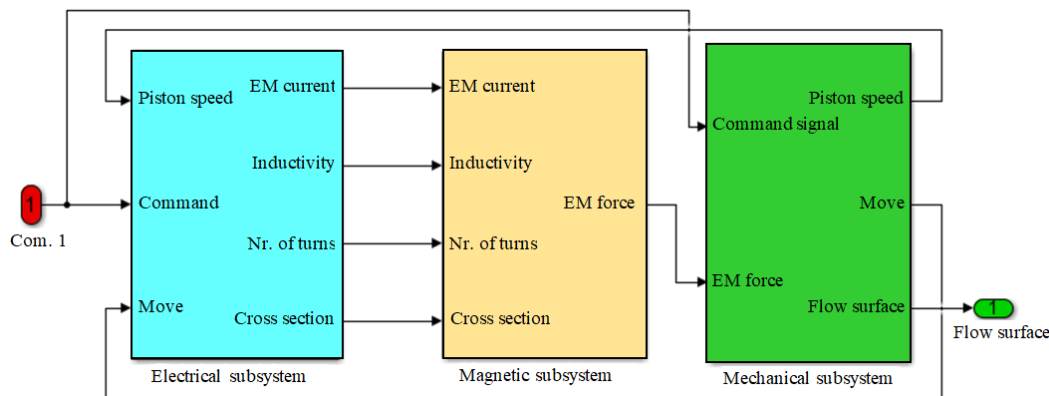


Figure 3 – Electromagnetic switch model

The mechanical subsystem consists of the spring and the anchor, whose movement changes the flow area through which air flows. The flow area represents the output of this subsystem, and consequently, the input quantity for the flow model calculations. For the purposes of analyzing phenomena in the switch, the displacement of the piston is also considered as the output of the mechanical subsystem.

The movement of the anchor is influenced by the electromagnetic force, the spring force, frictional forces, and forces arising from the pressure difference on different sides of the switch piston (Fig. 4).

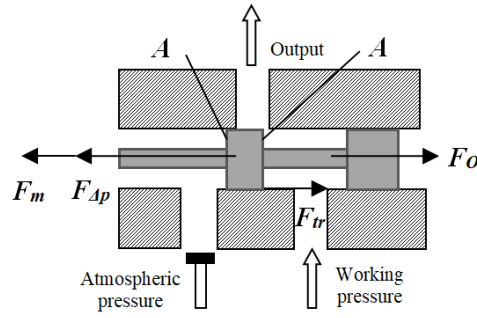


Figure 4 – Forces affecting the anchor movement

The equilibrium equation of the forces acting on the anchor is:

$$m_k \ddot{x}_k = F_m + F_{Ap} - F_{tr} - F_O \quad (1)$$

Where:

F_m – electromagnetic force,

F_{Ap} – force due to the pressure difference,

F_{tr} – frictional force,

F_O – spring force.

The force F_{Ap} arises due to the pressure difference that occurs on different sides of the switch piston:

$$F_{Ap} = A(p_g - p_d) \quad (2)$$

Where:

A – area on which the pressure difference acts,

p_g, p_d – upper and lower air pressures.

In the case of a different pressure arrangement, the force F_{Ap} in equation (2) will have the opposite sign.

The simplified expression for the frictional force is:

$$F_{tr} = B \cdot \dot{x}_k \quad (3)$$

Where:

B – coefficient of viscous friction.

The spring force is defined by the following equation:

$$F_O = F_{pn} + k_p x_k \quad (4)$$

Where:

F_{pn} – preload force of the spring,

k_p – spring stiffness,

x_k – anchor displacement.

In the general case, the change in the flow area is a function of the anchor displacement. The simplest assumption would be that the change occurs linearly [10]. In the literature [11], it is assumed that the conversion of anchor displacement follows a function represented in the form of a lookup table. In this example, it is assumed that the flow area changes according to the following relationship:

$$A_v = k \cdot x_k \quad (5)$$

Where:

$$k = \frac{A_0}{x_0} - \text{constant,}$$

A_0 – maximum opening area,

x_0 – maximum piston displacement.

The model of the mechanical subsystem is shown in Figure 5. The input quantities for the subsystem are the electromagnetic force, constants, the mass of the anchor, and the force due to the pressure difference, while the outputs are the displacement, velocity of the anchor, and the fluid exhaust area [12].

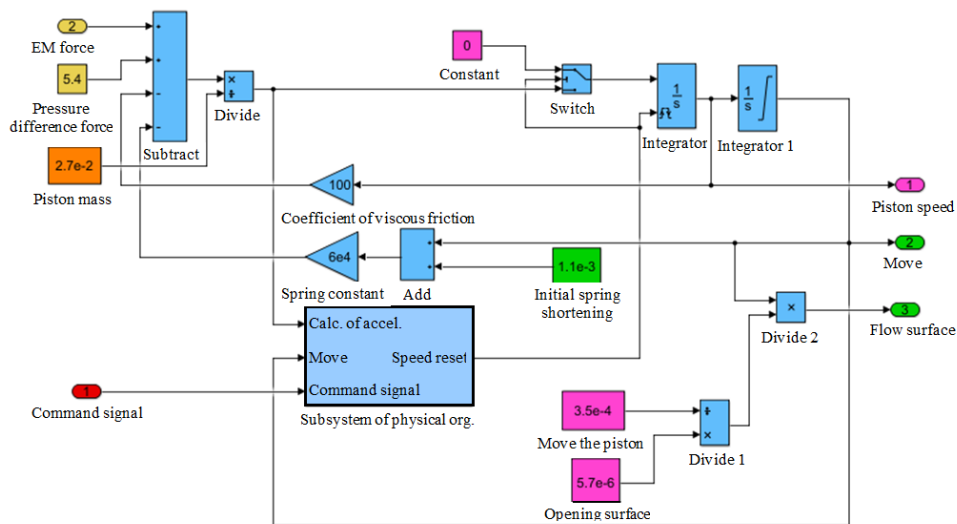


Figure 5 – Mechanical subsystem model

The magnitude of the spring force affects the switching on and off time of the switch. Figure 6 shows families of curves illustrating the dependence of the coil current and the anchor displacement on the spring stiffness.

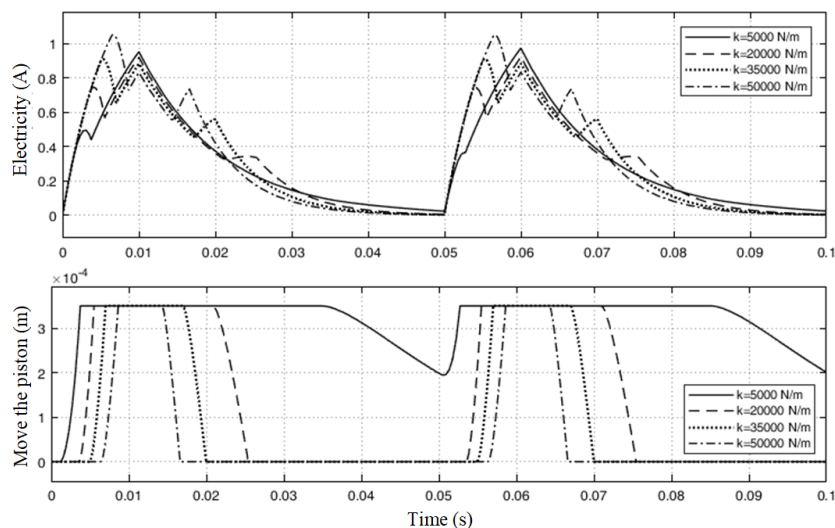


Figure 6 – Simulation results showing the effect of spring force magnitude

Based on the obtained simulation results, it can be concluded that increasing the spring force leads to an increase in the switching-on time (anchor attraction position), but also to a decrease in the switching-off time (anchor return).

By using the model of the electromagnetic switch and repeating the simulations with different values of spring stiffness, anchor mass, port area, supply voltage, etc., an analysis can be made of their impact on the output quantities, depending on different switch characteristics according to various criteria.

The analysis of this example identified the effects of the two most important factors on the switch's response time: the spring force and the supply voltage of the electromagnet (as presented in the literature [12]).

4. CONCLUSION

Mathematical models in mechanics enable complex physical processes and systems to be translated into understandable mathematical expressions that can be analyzed, solved, and used to predict behavior. Without these models, it would not be possible to develop modern technologies, engineering systems, or scientific theories. They are crucial for understanding the world around us, from everyday mechanical systems to the most complex research.

In this specific example of the electromagnetic switch, the importance of mathematical modeling can be seen. Due to the complexity and nonlinearity of the model, numerical modeling is most suitable, where a simulation model was developed in Simulink based on the mathematical model, and simulations were conducted accordingly. The application of mathematical modeling in technical systems is demonstrated in such a way that the formulated mathematical model is as general as possible, and by selecting parameters and making minor modifications, it can be used for various purposes.

5. REFERENCES

- [1] Arnol'd, V. I., Kozlov, V. V., Neishtadt, A. I., & Iacob, I. (2006). Mathematical aspects of classical and celestial mechanics (Vol. 3, pp. XIII-505). Berlin: Springer.
- [2] Arnol'd, V. I. (2013). Mathematical methods of classical mechanics (Vol. 60). Springer Science & Business Media.
- [3] Neittaanmäki, P., Repin, S., & Tuovinen, T. (Eds.). (2016). Mathematical modeling and optimization of complex structures (Vol. 40). New York: Springer.
- [4] Autor Velten, K., Schmidt, D. M., & Kahlen, K. (2024). Mathematical modeling and simulation: introduction for scientists and engineers. John Wiley & Sons.
- [5] Haupt, P. (1993). On the mathematical modelling of material behavior in continuum mechanics. *Acta Mechanica*, 100(3), 129-154.
- [6] Sofonea, M., & Matei, A. (2012). Mathematical models in contact mechanics (No. 398). Cambridge University Press.
- [7] Tayler, A. B. (2001). Mathematical models in applied mechanics (Vol. 4). Oxford University Press.
- [8] Haberman, R. (1998). Mathematical models: mechanical vibrations, population dynamics, and traffic flow. Society for Industrial and Applied Mathematics.
- [9] Tehrani, M. J., (2008), Pressure Control of a Pneumatic Actuator Using On/Off Solenoid Valves, Masters' Degree Project, Stockholm, Sweden, May 2008.
- [10] Taghizadeh, M., Ghaffari, A., Najafi F., (2009), Modeling and identification of a solenoid valve for PWM control applications, *C. R. Mecanique* 337 (2009), pp 131-140.
- [11] Youssry, M., Elmayyah, W., Mabrouk, M., Mahgoub, H., (2016), Parametric study of a low cost pneumatic system controlled by on/off solenoid valves, *IJRET International Journal of Research in Engineering and Tehnology*, Vol 05, pp 184-193.
- [12] Čajetinac, S., (2012). A contribution to the development of a decision-making model for the selection of electro-pneumatic control. Doctoral dissertation, University of Novi Sad, Faculty of Technical Sciences, Novi Sad.